THE EFFECT OF PLY CURVATURE ON ACOUSTIC WAVE PROPAGATION IN COMPLEX COMPOSITE STRUCTURES

R. A. Kline* and R. S. Gilmore
GE CRD
Manufacturing Technology Lab
Schenectady, NY 12301

INTRODUCTION

To date, most high performance composite materials have been fabricated in the form of relatively thin panels of uniform thickness, parallel front and back surfaces and plies parallel to these surfaces. For such panels, with only moderate curvature, analyses assuming plane wave propagation can treat the plies as being piecewise flat with curvature effects neglected. However, as composites become thicker with nonparallel surfaces and nonparallel plies, these simplified analyses must be abandoned. Depending on the extent of the nonuniformity in ply orientation, ply curvature may introduce a considerable amount of ray bending in the propagation of acoustic waves through composites with complex geometries and ply orientations. Accordingly, a mathematical framework is needed to treat this problem so that efficient, cost-effective inspection procedures may be designed and optimized.

In this work, a two-dimensional computer algorithm is developed to model acoustic wave propagation in composites which exhibit material property inhomogeneity due to ply curvature. The part is broken down into a rectangular grid of cells, each representing the local material properties. The baseline (straight ply) material properties are assumed to be orthotropic and ply curvature is introduced via a local ply orientation angle and standard transformation laws for the elements of the stiffness tensor. A ray propagation model, based on the Christoffel equation (incorporating the proper corrections for beam skew) is used to model wave propagation within each cell. Snell's law is used to represent refraction phenomena at the ply interfaces. The ray tracing part of the algorithm launches an acoustic wave in the medium and tracks it as it progresses from cell to cell. However, due to the large number of cells, the ray tracer concentrates on a single mode of propagation and ignores mode conversion at each interface. In this way, one can determine the transit times and path of travel of acoustic waves in complex composite structures. These calculations are very important if the inhomogeneities causing ultrasonic reflections are to be correctly located and analyzed.

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* On sabbatical leave from the School of AME, University of Oklahoma

Review of Progress in Quantitative Nondestructive Evaluation, Vol. 16
Edited by D.O. Thompson and D.E. Chimenti, Plenum Press, New York, 1997

BACKGROUND

The use of acoustic ray tracers to model wave propagation in heterogeneous media is not new. A variety of methods have been developed for this purpose. Typically, individual ray tracers are developed from Snell's law (modified for a continuum) or in a variational calculus form based on Fermat's principle. Early work describing ray bending in the earth was summarized by Bullen [1]. More recently,Lyttle and Dynes [2] used a local form of Snell's law to track acoustic waves in geological media i.e.

$$\frac{\sin(\alpha + d\alpha)}{v(x+dx)} = \frac{\sin(\alpha)}{x(x)} \qquad (1)$$

Berryman [3] developed a Fermat's principle ray tracer in a nonhomogeneous medium. Since we may represent the travel time (T) between the source (s) and receiver (r) along any ray path y(x) as

$$T(y) = \int_s^r m(x,y)\,ds \qquad (2)$$
$$where\ m(x,y) = slowness$$

the actual path is the one which yields a minimum in the transit time. The problem is solved using standard variational techniques. Later, Wang and Kline [4] extended this approach to anisotropic media. Alternatively, finite difference codes can be utilized for a more complete description of wave propagation features. Instead of concentrating on a single ray and tracking its progress from source to receiver, the finite difference approach, since it represents a full field solution to the equations of motion, allows one to launch a fan of rays simultaneously in multiple directions. Harker and Ogilvy [5] used this approach to examine material inhomogeneities in isotropic media. Kline [6] introduced anisotropy into this formulation and used it for tomographic reconstruction.

THEORY

The Christoffel equation which governs acoustic wave propagation in anisotropic media is given by

$$(C_{ijkl} l_j l_l - \rho v^2 \delta_{ik})\alpha_k = 0 \qquad (3)$$

where

$$C_{ijkl} = components\ of\ stiffness\ tensor$$
$$l_i = components\ of\ wave\ normal$$
$$\alpha_i = components\ of\ particle\ displacement$$
$$\rho = density$$
$$v = phase\ velocity$$
$$\delta_{ik} = components\ of\ identity\ tensor$$

For any given propagation direction l, this yields an eigenvalue problem with three possible phase velocities v, each associated with a polarization vector α. Unlike isotropic media where pure modes propagate and α and l are either parallel longitudinal) or perpendicular (transverse) to one another, in anisotropic media α and l are generally neither parallel or perpendicular to one another. The bulk waves which propagate will not be pure modes but rather will have some of the character of both longitudinal and transverse vibration. They are referred to as being quasilongitudinal and quasitransverse waves. A further complicating factor is beam skew which is commonly observed in anisotropic media. This refers to the fact that what one measures, energy, will not necessarily propagate in the direction of the wave normal at the phase velocity. Energy propagation is governed by the following equation [7]

$$S_j = \frac{C_{ijkl}\alpha_i\alpha_k l_l}{\rho v} \qquad (4)$$

where

$$\underset{\sim}{S} = energy\ propagation\ vector$$

$$\left|\underset{\sim}{S}\right| = group\ velocity$$

$$\underset{\sim}{S} \cdot \underset{\sim}{l} = v$$

The effects of material inhomogeneity and anisotropy must be carefully taken into account in any accurate ray tracing model.

WAVE PROPAGATION MODEL

A schematic diagram of the wave propagation model is presented in Figure 1. Here we model the material properties as being locally orthotropic, with the symmetry axes

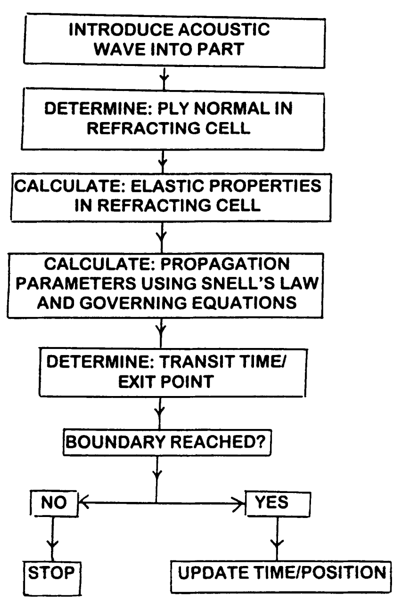

Figure 1 Schematic diagram of ray tracing algorithm.

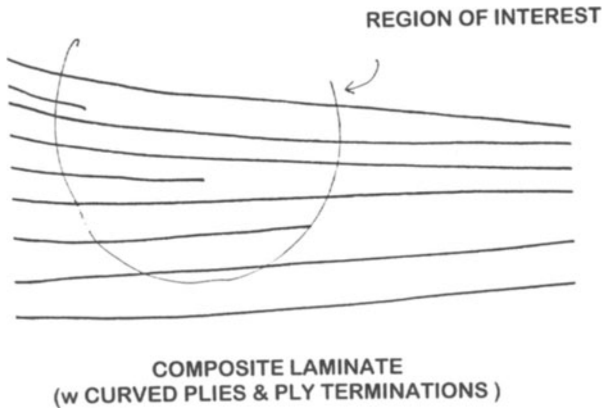

Figure 2 Ply curvature in transition region.

(x'- y')being rotated from the specimen coordinate axes (x - y) due to the presence of curvature (see Figure 2). It is assumed that the material is locally uniform in the z direction; hence the ply curvature can be described by a single rotation angle. Since the rotation angle varies from point to point within the specimen, a look up table is required so that the proper elastic moduli are input into the ray tracer algorithm. Ply terminations (also shown in Figure 2) have been neglected at this stage of model development. Models for incorporating this effect are currently under investigation. A diagram illustrating the variation in ply normal with position for a typical section of interest is shown in Figure 3. The standard equation for tensor rotation is used to

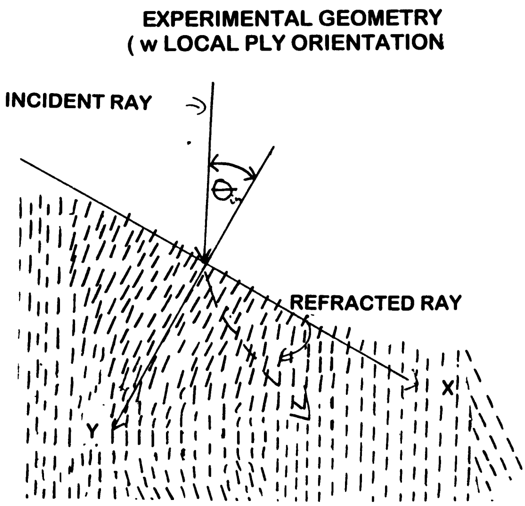

Figure 3 Ply normals in transition region.

modify the local elastic moduli

$$C_{ijkl} = a_{i\alpha}\, a_{j\beta}\, a_{k\gamma}\, a_{l\delta}\, C_{\alpha\beta\gamma\delta} \tag{5}$$

where

$$a_{ij} = components\ of\ rotation\ matrix$$

at each point in the domain.

For modeling purposes, the domain is broken down into a series of square cells, each of which is associated with its proper ply orientation angle in the look up table. When an elastic wave (quasilongitudinal or quasitransverse) is introduced into a new cell (see Figure 4), the rotation angle is read from the look up table and the moduli calculated. Snell's law is then used to determine the wave normal in the new cell. It should be noted that application of Snell's law requires the solution of the Christoffel equation since the material is anisotropic and, unlike isotropic media, the phase velocity is a function of the direction of propagation. In general, there will be three possible solutions to this problem; corresponding to each of the possible modes of propagation. Here, we restrict our attention to a single mode of interest and ignore the mode conversion effects which are likely to be quite small for the gradual changes in ply curvature in these samples. Next, the direction and velocity of energy propagation is calculated. This allows us to determine the point where the ray will exit from the cell, the coordinates of the next cell along the ray path and the transit time through the material. The process is repeated, cell by cell, until the boundary is reached. In this way, the path of any acoustic ray in the sample can be determined. This process is illustrated in Figure 4.

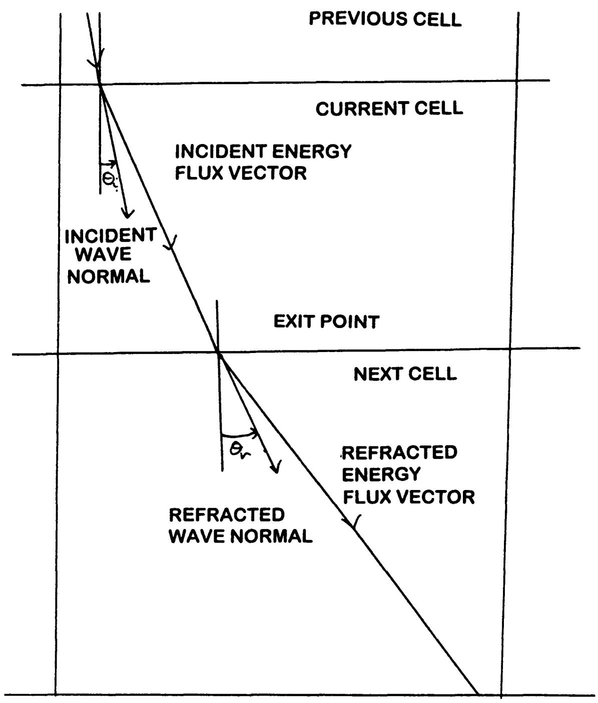

Figure 4 Geometry for discrete cell ray tracer .

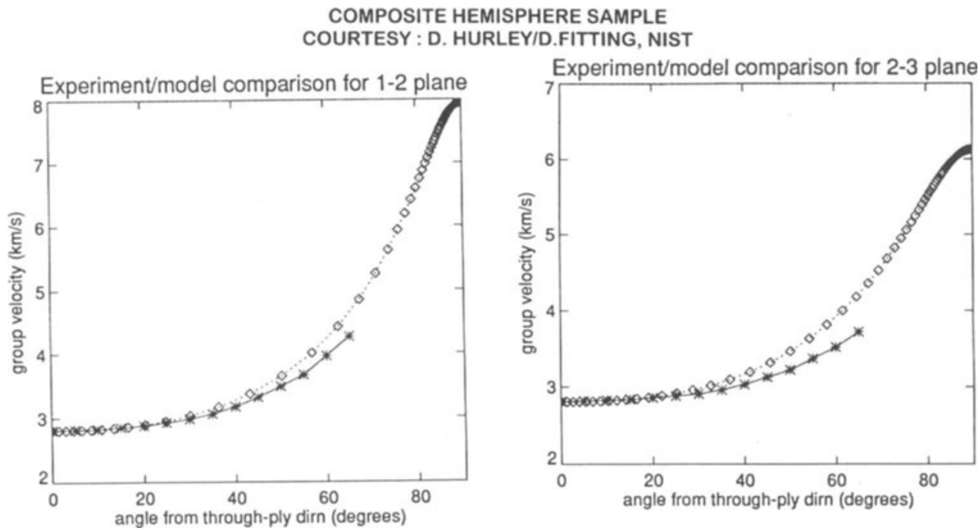

Figure 5 Results for hemispherical sample a) 12 plane b) 23 plane.

RESULTS AND DISCUSSION

In order to validate the material properties used in this algorithm, it was necessary to examine acoustic wave propagation along multiple ray paths in samples of known ply orientation. Model results were compared with experimental data obtained previously on hemispherical samples of similar composition [8]. The hemispherical geometry is particularly suited to this task as any possible ray path can be readily accommodated experimentally. Table 1 presents the elastic moduli for a single, unidirectionally reinforced ply of this material as determined from mechanical testing (1 being the fiber direction)

Table 1 Mechanical Test Results - Single Ply

MODULUS	TEST MODE	MEASURED VALUE
E1	Tension	157.9 Gpa
E1	Compression	144.2 Gpa
E2	Tension	8.26 Gpa
E2	Compression	9.49 Gpa
E3	Tension	9.15 Gpa
E3	Compression	12.45 Gpa
G12		5.44 Gpa
G13		4.33 Gpa
G23		3.02 Gpa
v12		0.32
v13		0.35
v23		0.37

Table 2 Orthotropic moduli

$$C_{ij}=\left(\begin{array}{cccccc}92.55 & 6.76 & 24.48 & 0 & 0 & 0\\ 6.76 & 11.87 & 6.63 & 0 & 0 & 0\\ 24.48 & 6.63 & 43.09 & 0 & 0 & 0\\ 0 & 0 & 0 & 3.81 & 0 & 0\\ 0 & 0 & 0 & 0 & 22.00 & 0\\ 0 & 0 & 0 & 0 & 0 & 4.66\end{array}\;0\right) GPa$$

Using the values in Table 1, averaging the tensile and compressive response and knowing lay-up sequence for the specimen, the reduced stiffnesses shown in Table 2 are obtained from classical lamination theory [9]. Using these values, the variation in group velocity with propagation direction was calculated for the 12 and 23 planes. These results were compares with the data measured on the hemispherical samples. A comparison of those results is presented in Figure 5. Good agreement between theory and experiment was observed. Using this data, it was then possible to use the ray tracing algorithm to track the path of acoustic rays in the sample. Figure 6 presents a typical result for a 1" by 1" section of a region with pronounced ply curvature. The deviation from straight line ray propagation, even over this very short distance, should be noted. With this approach, it is possible to accurately track acoustic ray paths in any desired region of the composite. This information is critically important in precisely determining the location of potential defects in conventional NDE testing.

SUMMARY

- An ultrasonic technique has been described to track acoustic wave propagation in composites with ply curvature.

- The model is based on a sequential ray tracer. The domain is discretized and the ray path determined by repeated application of Snell's law at each interface.

- The material is modeled as being orthotropic. Ply curvature effects are introduced from a table which gives ply orientation, position by position, throughout the sample. A tensor transformation is used to account for the coordinate system rotation.

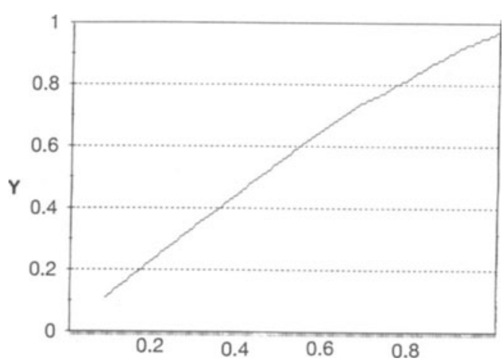

Figure 6 Ray tracer results for transition region.

- Ray tracing results have been presented for wave propagation in a high curvature region of a thick composite structure.

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